Table 1 Plateau propellants with aromatic lead compounds

NC[a]	NG[a]	TA[a]	NDPA[a]	Salt	H_{ex}, cal/g	π, %/°F
30	67	...	3	0	1440	1.0
30	67	...	1	2 (benzoate)	1450	0.57
35	60	...	1	4 (phthalate)	1380	0.60
45	52	...	1	2 (benzoate)	1350	0.55
50	47	...	1	2 (benzoate)	1270	0.60
50	43	3	1	3 (tannate)	1190	0.60
50	43	4	1	2 (benzoate)[b]	1180	0.60
50	43	4	1	2 (benzoate)	1170	0.70
58.5	37	...	2.5	4 (salicylate)	1120	0.37
59.5	33	4	2.5	2 (trimesate)	1030	0.41
58.5	27	10	2.5	2 (thiosalicylate)	855	0.06
58.5	27	10	2.5	2 (anthranilate)	850	0.14
58.5	27	12	2.5	0	850	0.80

[a] NC = nitrocellulose of 12.6% N, NG = nitroglycerine, TA = triacetin, and NDPA = 2-nitrodiphenylamine.
[b] Contains 0.2 carbon black (Carbolac I) on added basis.

Table 2 Burning rates in the plateau for a composition like that of Fig. 2, obtained at 0.5% carbon black

	Rate at 2000 psi and 25°C, in./sec
Excello 0.030 μ average particle diameter	0.52
Neo Spectra Mk I 0.010 μ (Binney & Smith Co.)	0.57
Kosmos F-4 0.008 μ (United Carbon Co., Inc.)	0.58
Carbolac I 0.009 μ (Godfrey L. Cabot, Inc.)	0.64

Table 3 Burning rates obtained with 0.5% pigment in the standard formulation

	Rate at 2000 psi and 25°C, in./sec
Excello 0.030 μ	0.52
Graphite 1.0 μ	0.48
Hydrated alumina 0.5 μ	0.48
Acetylene black 0.05 μ	0.46
Phthalocyanine	0.37
Titanium oxide 0.4 μ	0.31
Magnesium oxide	0.31
Levigated alumina 3 μ	0.31
None	0.31

(see Tables 2 and 3). Although the graphite has a nominal particle diameter 1 μ, its very thin leaflets afford far more surface than the denser TiO_2 at 0.4 μ with a more compact particle. At higher concentrations TiO_2 gives plateau rate boosts (to 0.44 in./sec at 1%).

A plot of plateau rate vs particle size is shown in Fig. 3. If actual specific surface data were available, the smoothness of fit of the points on the curve might be improved. At particle sizes much above 0.1 μ, little or no rate boost is observed and the pigment loses ballistic effect.

Combining high rate promoters, i.e., aromatic lead salts, high energy, and small particle carbon black has led to the highest plateau burning rates encountered in nitrocellulose systems.[6] An example containing 4% lead 2,4-dihydroxybenzoate and 0.4% carbolac I at an energy of 1350 cal/g showed a rate of 1.1 in./sec at 1100 psi with plateau ballistics. Omission of carbon black drops the rate to 0.8 in./sec at this pressure, and decrease of either lead salt or carbon black leads to more or less loss in burning rate. Substantial increase of concentration of either of these coolant ingredients results in decreased burning rates. One observes a maximum burning rate system, in a given range of salt and carbon black concentrations, which is very sensitive to mixing and requires good process control. Any local excess or deficiency of either or both of the minor constituents leads to burning rate decrease.

References

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Torsion of an Elastic Cylinder of Unsymmetrical Aerofoil Cross Section

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In this note the method developed by Deutsch[1] is applied for solving the Saint-Venant torsion problem of an elastic cylinder whose cross section resembles an unsymmetrical aerofoil. The method consists of mapping the cross section of the beam conformally on a unit semicircle and solving the resulting Dirichlet problem.

1. Basic Equations

Let

$$Z = w(\zeta) \tag{1.1}$$

be the function that maps conformally a region S of a Z plane ($Z = x + iy$) on the semicircle $|\zeta| \leqslant 1$, $\eta \geq 0$ of a ζ plane ($\zeta = \xi + i\eta$). Let $f(z)$ be the complex-torsion function[2] of the region S and let $\phi(\zeta) = \phi[w(\zeta)]$ be the same complex torsion function expressed in terms of the variable ζ.

Then the complex torsion function is given by[1]

$$\phi(\zeta) = \frac{1}{2\pi} \int_\gamma w(\sigma)\bar{w}\left(\frac{1}{\sigma}\right)\left(\frac{1}{\sigma - \zeta} + \frac{1}{\sigma - (1/\zeta)}\right) d\sigma + \frac{1}{2\pi}\int_{-1}^{+1} w(\xi)\bar{w}(\xi)\left(\frac{1}{\xi - \zeta} + \frac{1}{\xi - (1/\zeta)}\right) d\xi \tag{1.2}$$

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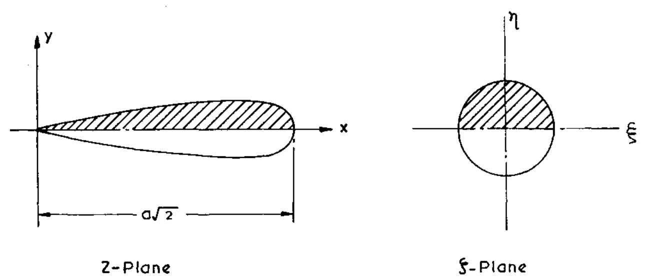

Fig. 1 a) Cross section of the cylinder; b) mapping.

where γ denotes the semicircumference $|\zeta| = 1$, $\eta \geq 0$ and $\sigma = e^{i\theta}$ is a point on γ.

The torsional rigidity D is given by

$$D = \mu[I + L + \bar{L}] \tag{1.3}$$

where μ is the modulus of rigidity, I the polar moment of inertia of the region S with respect to the origin of the coordinate axes. Further,

$$I = -\frac{1}{4} i \int_\gamma \bar{w}^2\left(\frac{1}{\sigma}\right) w(\sigma) dw(\sigma) - i\frac{1}{4}\int_{-1}^{+1} \bar{w}^2(\xi) w(\xi) dw(\xi) \tag{1.4}$$

$$L = -\frac{1}{4}\int_\gamma \phi(\sigma) d\left\{w(\sigma)\bar{w}\left(\frac{1}{\sigma}\right)\right\} - \frac{1}{4}\int_{-1}^{+1} \phi(\xi) d\{w(\xi)\bar{w}(\xi)\} \tag{1.5}$$

If the mapping function $w(\zeta)$ can be continued analytically into the semicircle $|\zeta| \leq 1$, $\eta < 0$ then (1.2, 1.4, and 1.5) can be reduced to

$$\phi(\zeta) = iw(\zeta)\bar{w}(\zeta) + \frac{1}{2\pi}\int_\gamma w(\sigma) \times \left\{\bar{w}\left(\frac{1}{\sigma}\right) - \bar{w}(\sigma)\right\}\left\{\frac{1}{\sigma - \zeta} + \frac{1}{\sigma - (1/\zeta)}\right\} d\sigma \tag{1.6}$$

$$I = -\frac{1}{4} i \int_\gamma \left\{\bar{w}^2\left(\frac{1}{\sigma}\right) - \bar{w}^2(\sigma)\right\} w(\sigma) dw(\sigma) \tag{1.7}$$

$$L = -\frac{1}{4}\int_\gamma \phi(\sigma) d\left[w(\sigma)\left\{\bar{w}\left(\frac{1}{\sigma}\right) - \bar{w}(\sigma)\right\}\right] \tag{1.8}$$

which are sometimes more convenient for computations.

2. Cross Section and Complex Torsion Function

The mapping function

$$Z = w(\zeta) = a(1 + \zeta)^{1/2} (a \text{ real}) \tag{2.1}$$

maps conformally one-half of a loop of Bernoulli's Lemniscate in the z plane on a unit semicircle $|\zeta| \leq 1$, $\eta > 0$ of the ζ plane (Fig. 1).

We consider the torsion of this section in the z plane which resembles an unsymmetrical aerofoil. From (2.1) we have

$$w(\sigma) = a(1 + \sigma)^{1/2} \qquad \bar{w}(\bar{\sigma}) = a\left(\frac{1 + \sigma}{\sigma}\right)^{1/2}$$

$$dw(\sigma) = \frac{a(1 + \sigma)^{-1/2}}{2} d\sigma \qquad w(\sigma)\bar{w}(\bar{\sigma}) = \frac{a^2(1 + \sigma)}{\sigma^{1/2}} \tag{2.2}$$

$$\bar{w}(\sigma)\, w(\sigma) = a^2(1 + \sigma)$$

Taking into account (2.2) from (1.6), the complex torsion function, apart from a nonessential constant, is

$$\phi(\zeta) = \frac{a^2(1 + \zeta)}{2\pi}\left[\pi i - \frac{1 + \zeta}{\zeta}\log\frac{1 + \zeta}{1 - \zeta} + \frac{2}{\zeta^{1/2}}\log\frac{1 + \zeta^{1/2}}{1 - \zeta^{1/2}}\right] \tag{2.3}$$

In terms of the variable z, the complex torsion function can be written

$$f(z) = \frac{z^2}{2\pi}\left[\pi i - \frac{z^2}{z^2 - a^2}\log\frac{z^2}{2a^2 - z^2} + \frac{2a}{(z^2 - a^2)^{1/2}}\log\frac{a + (z^2 - a^2)^{1/2}}{a - (z^2 - a^2)^{1/2}}\right] \tag{2.4}$$

3. Torsional Rigidity and Complex Shear Stress

The polar moment of inertia I of the cross section is obtained by substituting (2.2) into (1.7). It is found that

$$I = \pi a^4/8 \tag{3.1}$$

The calculation for L is more laborious in that it involves the complex torsion function. The integrals were calculated by expanding the functions in powers of σ. It reduces to

$$L = \frac{a^4}{16\pi}[\pi^2 - 4\pi + 116] + \frac{ia^4}{12\pi}[4 - \pi] \tag{3.2}$$

where we have taken into account the fact that

$$\sum_{n=1}^{\infty}\frac{1}{(2n - 1)^2} = \frac{\pi^2}{8} \qquad \sum_{n=1}^{\infty}\frac{1}{(4n - 1)^2} = \frac{1}{2}$$

$$\sum_{n=1}^{\infty}\frac{1}{(4n - 5)(4n - 3)(4n - 1)(4n + 1)} = -\frac{\pi}{48}$$

$$\sum_{n=1}^{\infty}\frac{n}{(2n + 1)(4n - 1)(4n + 1)} = \left(\frac{\pi}{48} - 1\right)$$

$$\sum_{n=1}^{\infty}\frac{1}{(2n - 1)(4n - 1)(4n - 3)(4n - 5)(4n + 1)} = -\frac{1}{36}$$

Hence,

$$L + \bar{L} = \frac{a^4}{8\pi}[\pi^2 - 4\pi + 116]$$

Hence, the torsional rigidity from (1.3) is

$$D = (\mu a^4/4\pi)[\pi^2 - 2\pi + 58] \tag{3.3}$$

The complex shear stress is given by[2]

$$\tau_{zx} - i\tau_{zy} = \mu\alpha\left[\frac{\phi'(\zeta)}{w'(\zeta)} - i\bar{w}(\bar{\zeta})\right] = \mu\alpha\left\{\frac{a(1 + \zeta)^{1/2}}{\pi}\left[\pi i - \frac{\zeta^2 - 1}{\zeta^2}\log\frac{1 + \zeta}{1 - \zeta} - \frac{1 - \zeta}{\zeta^{3/2}}\log\frac{1 + \zeta^{1/2}}{1 - \zeta^{1/2}}\right] - ia\left(\frac{1 + \zeta}{\zeta}\right)^{1/2}\right\} \tag{3.4}$$

or, in terms of z,

$$\tau_{zx} - i\tau_{zy} = \mu\alpha\left[i(z - \bar{z}) - \frac{z^3(z^3 - 2a^2)}{\pi(z^2 - a^2)^2}\log\frac{z^2}{2a^2 - z^2} - \frac{az(2a^2 - z^2)}{\pi(z^2 - a^2)^{3/2}}\log\frac{a + (z^2 - a^2)^{1/2}}{a - (z^2 - a^2)^{1/2}}\right] \tag{3.5}$$

where α is the twist per unit length.

References

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